

Analysis of Coupled RC Shear/Core Walls by Macro Model

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Abstract

A modified macro numerical model for coupled RC shear/core walls is proposed to better consider the deformation compatibility between the coupling beam and the wall element and to incorporate the out-of-plane effect of the wall panel. In this model, the three-dimensional macro shear wall element model with distributed shear springs and out-of-plane freedom is adopted to simulate the wall unit; a one-dimensional line element consisting of three sub-units in series based on the vertical deformation compatibility is adopted to simulate the coupling beam. The case study shows that a numerical model based on the vertical deformation compatibility results in more accurate simulation of the mechanical behavior of the RC coupled shear wall. The modified model developed in this study has the advantages of fast calculation and high precision, which makes the model appropriate for engineering application.

Keywords: coupled RC shear wall; deformation compatibility; macro model; beam-wall joint rotation

1. Introduction

RC shear walls have been widely used in tall buildings in earthquake prone areas because of their high lateral stiffness and strength. In recent decades, a large number of numerical models have been developed to simulate the nonlinear responses of RC shear walls, which can be classified into two categories in general: micro model and macro model. The finite element model is the most representative micro model.

For engineering practice purposes, the macro model is desirable because of its capability to reasonably simulate the main characteristics of shear walls using one element for one story to simplify modeling and reduce computational efforts. Several different types of macro models for shear walls have been developed, ranging from a simple one-dimensional beam element to a complicated three-dimensional wall panel model. At first, the equivalent beam model (EBM) was popularly applied in practice. However, this model has a fatal defect because of the assumption that rotations always occur around the centroidal axis and that the fluctuation of the cross-section neutral axis is disregarded even if the wall gets into an inelastic state. The multi-vertical-line-element model (MVLEM) remedying the EBM defect was proposed

by Valcano *et al.* (1988). In this model, the shear wall was represented by a set of nonlinear vertical and horizontal springs connected by two rigid beams at the top and the bottom. MVLEM has been widely used in the numerical analysis of RC shear walls. Some modified MVLEMs have been developed to improve the prediction of the nonlinear behavior of RC shear walls (Linde and Bachmann, 1994; Jiang *et al.*, 2003). However, for simplicity, the out-of-plane effect of the wall panel is ignored in a MVLEM.

Many shear walls contain one or more vertical rows of openings, resulting in so-called "coupled shear walls". In coupling beam models, it has been unanimously assumed that the rotation of the coupling beam at the beam-wall joint is equal to that of the horizontal fiber at the wall side of the joint (horizontal deformation compatibility, for short). In almost all of the macro models developed for RC coupled shear walls, the stiffness of the coupling beam was derived on the basis of this assumption. However, the adoption of this definition may lead to deformation incompatibility between the beam and the wall unit because the beam-wall boundary is actually a vertical edge and the rotation of the vertical fiber at the edge is not necessarily equal to that of the horizontal fiber there, as shown in Fig.1.(a) (Kwan, 1993). Such an incompatibility will cause errors in the stiffness of the coupling beam and the joint rotation, especially in those cases where the wall unit is subjected to significant shear deformation. To ensure compatibility between the beam and the wall element, the rotation degree of freedom (DOF) at the beam-wall joint should be taken as the rotation of the beam-wall boundary, i.e., the rotation of the vertical fiber at the

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edge of the wall (vertical deformation compatibility, for short), as shown in Fig.1.(b).

To better predict the nonlinear responses of RC coupled shear/core walls, a modified macro numerical model based on a MVLEM is developed in this study, to better consider the deformation compatibility between the coupling beam and the wall element and incorporate the out-of-plane effect of the wall panel.

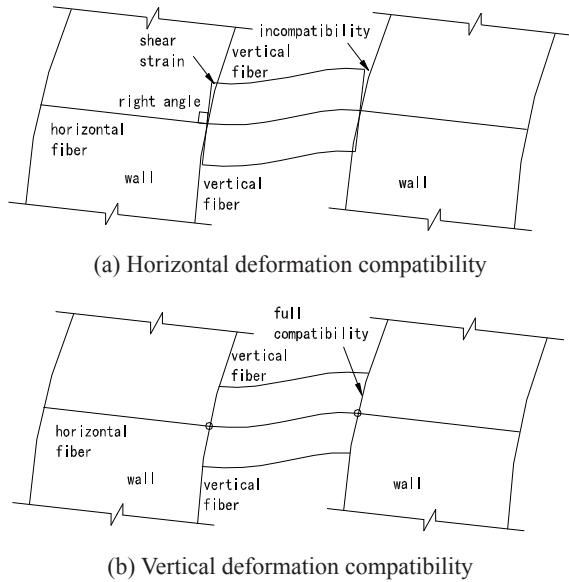


Fig.1. Definition of Rotation DOF at the Beam-wall Joint

2. Coupling Beam Model

2.1 Element Stiffness in Local Coordinates

The one-dimensional line element consisting of three sub-units in series, as shown in Fig.2., i.e., the distributed plastic flexural unit, the bond-slip unit, and the shear unit, is adopted to simulate the flexural deformation, the slip of the longitudinal steel bar, and shear deformation of the coupling beam, respectively. The additional rotation caused by the slip of the longitudinal steel bar in the anchoring zone is represented by the rotational spring at the end. In the local coordinates, the equilibrium equation for the coupling beam can be expressed as

$$\begin{Bmatrix} F_{ix'} \\ F_{iy'} \\ M_{iz'} \\ F_{jx'} \\ F_{jy'} \\ M_{jz'} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ & & k_{33} & k_{34} & k_{35} & k_{36} \\ & & & k_{44} & k_{45} & k_{46} \\ & & & & k_{55} & k_{56} \\ & & & & & k_{66} \end{bmatrix} \begin{Bmatrix} u_{ix'} \\ u_{iy'} \\ \theta_{iz'} \\ u_{jx'} \\ u_{jy'} \\ \theta_{jz'} \end{Bmatrix} \quad (1)$$

Sym.

where $u_{ix'}$, $u_{iy'}$, $\theta_{iz'}$, are the displacement in the x, y direction and the rotation in the z direction at the i th node, respectively, and $u_{jx'}$, $u_{jy'}$, $\theta_{jz'}$, are the displacement in the x, y direction and the rotation in the z direction at the j th node, respectively.

Equation 1 can be abbreviated as

$$F_c' = K_c' d_c' \quad (2)$$

where K_c' is the element stiffness matrix in local coordinates, d_c' is the displacement vector, and F_c' is the force vector.

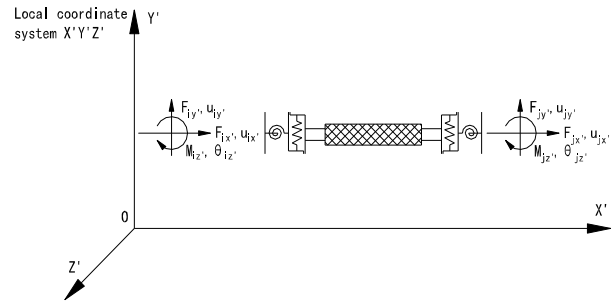


Fig.2. Numerical Model for Coupling Beam

2.2 Element Stiffness with Vertical Deformation Compatibility

1. Master to slave transformation matrix in local coordinates

When the rotation DOF of the beam-wall joint is defined by the vertical deformation compatibility, the displacement vector and force vector can be expressed as follows

$$d_z' = \{u_{1mx'}, u_{1my'}, u_{1nx'}, u_{2qx'}, u_{2qy'}, u_{2px'}\}^T \quad (3)$$

$$F_z' = \{F_{1mx'}, F_{1my'}, F_{1nx'}, F_{2qx'}, F_{2qy'}, F_{2px'}\}^T \quad (4)$$

The positive direction of each parameter is shown in Fig.3.

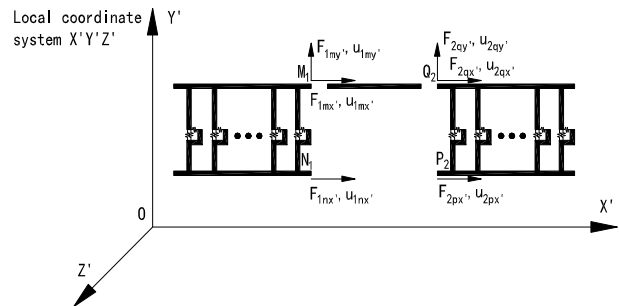


Fig.3. Beam Element Model with Vertical Deformation Compatibility

The deformation relationship between the beam and the wall element is given by the following expressions:

$$u_{ix'} = u_{1mx'}, \quad u_{jx'} = u_{2qx'} \quad (5)$$

$$u_{iy'} = u_{1my'}, \quad u_{jy'} = u_{2qy'} \quad (6)$$

$$\theta_{iz'} = \frac{u_{1mx'} - u_{1nx'}}{H_1}, \quad \theta_{jz'} = \frac{u_{2qx'} - u_{2px'}}{H_2} \quad (7)$$

where H_1 and H_2 are the height of the left wall and right wall units, respectively.

The master to slave displacement transformation equation in the local coordinates can be expressed as

$$\begin{Bmatrix} u_{ix'} \\ u_{iy'} \\ \theta_{iz'} \\ u_{jx'} \\ u_{jy'} \\ \theta_{jz'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ 1/H_1 & & -1/H_1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & 1/H_2 & & -1/H_2 \end{bmatrix} \begin{Bmatrix} u_{1mx'} \\ u_{1my'} \\ u_{1nz'} \\ u_{2qx'} \\ u_{2qy'} \\ u_{2qz'} \end{Bmatrix} \quad (8)$$

Equation 8 can be abbreviated as

$$d_c' = J d_z' \quad (9)$$

where J is the master to slave transformation matrix in local coordinates.

The following expressions can be derived by using the force synthesis and decomposition:

$$F_{ix'} = F_{1mx'} + F_{1nx'}, \quad F_{jx'} = F_{2qx'} + F_{2px'} \quad (10)$$

$$F_{iy'} = F_{1my'}, \quad F_{jy'} = F_{2qy'} \quad (11)$$

$$M_{iz'} = -F_{1nx'} H_1, \quad M_{jz'} = -F_{2px'} H_2 \quad (12)$$

The equation of the master to slave force transformation in the local coordinates can be expressed as

$$\begin{Bmatrix} F_{ix'} \\ F_{iy'} \\ M_{iz'} \\ F_{jx'} \\ F_{jy'} \\ M_{jz'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & -H_1 & & & \\ & & & 1 & & 1 \\ & & & & 1 & \\ & & & & & -H_2 \end{bmatrix} \begin{Bmatrix} F_{1mx'} \\ F_{1my'} \\ F_{1nx'} \\ F_{2qx'} \\ F_{2qy'} \\ F_{2px'} \end{Bmatrix} \quad (13)$$

The following equation can be derived by transforming the above expression:

$$\begin{Bmatrix} F_{1mx'} \\ F_{1my'} \\ F_{1nx'} \\ F_{2qx'} \\ F_{2qy'} \\ F_{2px'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & -1/H_1 & & & \\ & & & 1 & & 1/H_2 \\ & & & & 1 & \\ & & & & & -1/H_2 \end{bmatrix} \begin{Bmatrix} F_{ix'} \\ F_{iy'} \\ M_{iz'} \\ F_{jx'} \\ F_{jy'} \\ M_{jz'} \end{Bmatrix} \quad (14)$$

It is obvious that the matrix in Equation 14 is the transpose of conversion matrix J . Thus, it can be abbreviated as

$$F_z' = J^T F_c' \quad (15)$$

The stiffness matrix in the local main coordinates can be derived as follows:

$$F_z' = K_z' d_z' \quad (16)$$

$$K_z' = J^T K_c' J \quad (17)$$

2. Global to local transformation matrix of master degree of freedom

The displacement vector and force vector can be expressed as follows:

$$d_z = \{u_{1mx}, u_{1my}, u_{1mz}, u_{2qx}, u_{2qy}, u_{2qz}, u_{1nx}, u_{1ny}, u_{1nz}, u_{2px}, u_{2py}, u_{2pz}\}^T \quad (18)$$

$$F_z = \{F_{1mx}, F_{1my}, F_{1mz}, F_{2qx}, F_{2qy}, F_{2qz}, F_{1nx}, F_{1ny}, F_{1nz}, F_{2px}, F_{2py}, F_{2pz}\}^T \quad (19)$$

where u_{1mx} , u_{1my} , and u_{1mz} denote the displacement in the x, y, and z directions at the m th node on the left wall, respectively; F_{1mx} , F_{1my} , and F_{1mz} denote the force in the x, y, and z directions at the m th node on the left wall, respectively; u_{1nx} , u_{1ny} , and u_{1nz} denote the displacement in the x, y, and z directions at the n th node on the left wall, respectively; F_{1nx} , F_{1ny} , and F_{1nz} denote the force in the x, y, and z directions at the n th node on the left wall, respectively; u_{2qx} , u_{2qy} , and u_{2qz} denote the displacement in the x, y, and z directions at the q th node on the right wall, respectively; F_{2qx} , F_{2qy} , and F_{2qz} denote the force in the x, y, and z directions at the q th node on the right wall, respectively; u_{2px} , u_{2py} , and u_{2pz} denote the displacement in the x, y, and z directions at the p th node on the right wall, respectively; and F_{2px} , F_{2py} , and F_{2pz} denote the force in the x, y, and z directions at the p th node on the right wall, respectively.

Global to local displacement transformation equation of the master degree of freedom can be derived as follows:

$$T = \begin{bmatrix} \cos(\gamma) & 0 & -\sin(\gamma) & & & \\ 0 & 1 & 0 & & & \\ & & & \cos(\gamma) & 0 & -\sin(\gamma) \\ & & & 0 & 1 & 0 \\ & & & & & \cos(\gamma) & 0 & -\sin(\gamma) \end{bmatrix}_{6 \times 12} \quad (20)$$

Equation 20 can be abbreviated as

$$d_z' = T d_z \quad (21)$$

where T is the global to local transformation matrix of the master degree of freedom.

The stiffness matrix in global main coordinates can be derived as follows:

$$F_z = K_z d_z \quad (22)$$

$$K_z = T^T K_z' T \quad (23)$$

2.3 Element Stiffness with Horizontal Deformation Compatibility

1. Master to slave transformation matrix in local coordinates

When the rotation DOF of the beam-wall joint is defined by the horizontal deformation compatibility, the displacement vector and force vector can be expressed as follows:

$$d_z' = \{u_{1mx'}, u_{1my'}, u_{1qy'}, u_{2qx'}, u_{2qy'}, u_{2my'}\}^T \quad (24)$$

$$F_z' = \{F_{1mx'}, F_{1my'}, F_{1qy'}, F_{2qx'}, F_{2qy'}, F_{2my'}\}^T \quad (25)$$

The positive direction of each parameter is shown in Fig.4.

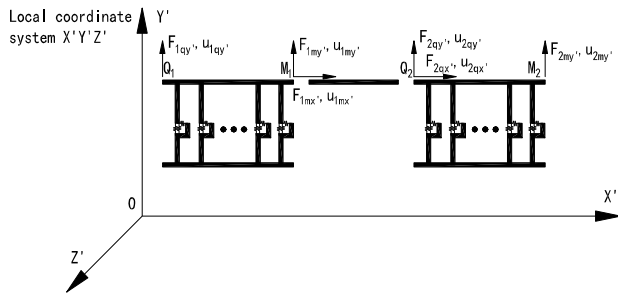


Fig.4. Beam Element Model with Horizontal Deformation Compatibility

The deformation relationship between the beam and the wall element is given by the following expressions:

$$u_{ix'} = u_{1mx'}, \quad u_{jx'} = u_{2qx'} \quad (26)$$

$$u_{iy'} = u_{1my'}, \quad u_{jy'} = u_{2qy'} \quad (27)$$

$$\theta_{iz'} = \frac{u_{1qy'} - u_{1my'}}{L_1}, \quad \theta_{jz'} = \frac{u_{2qy'} - u_{2my'}}{L_2} \quad (28)$$

where L_1 and L_2 are the width of the left wall and right wall, respectively.

The master to slave displacement transformation equation can be expressed in local coordinates as follows:

$$\begin{Bmatrix} u_{ix'} \\ u_{iy'} \\ \theta_{iz'} \\ u_{jx'} \\ u_{jy'} \\ \theta_{jz'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & -1/L_1 & 1/L_1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1/L_2 & -1/L_2 \end{bmatrix} \begin{Bmatrix} u_{1mx'} \\ u_{1my'} \\ u_{1qy'} \\ u_{2qx'} \\ u_{2qy'} \\ u_{2my'} \end{Bmatrix} \quad (29)$$

Equation 29 can be abbreviated as

$$d_c' = J d_z' \quad (30)$$

where J is the master to slave transformation matrix in local coordinates.

The following expressions can be derived by using the force synthesis and decomposition:

$$F_{ix'} = F_{1mx'}, \quad F_{jx'} = F_{2qx'} \quad (31)$$

$$F_{iy'} = F_{1my'} + F_{1qy'}, \quad F_{jy'} = F_{2qy'} + F_{2my'} \quad (32)$$

$$M_{iz'} = F_{1qy'} L_1, \quad M_{jz'} = -F_{2my'} L_2 \quad (33)$$

The master to slave force transformation equation in local coordinates can be expressed as follows:

$$\begin{Bmatrix} F_{ix'} \\ F_{iy'} \\ M_{iz'} \\ F_{jx'} \\ F_{jy'} \\ M_{jz'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & 1 & & & \\ & & L_1 & & & \\ & & & 1 & & \\ & & & & 1 & 1 \\ & & & & & -L_2 \end{bmatrix} \begin{Bmatrix} F_{1mx'} \\ F_{1my'} \\ F_{1qy'} \\ F_{2qx'} \\ F_{2qy'} \\ F_{2my'} \end{Bmatrix} \quad (34)$$

Transformation of the above expression is as follows:

$$\begin{Bmatrix} F_{1mx'} \\ F_{1my'} \\ F_{1qy'} \\ F_{2qx'} \\ F_{2qy'} \\ F_{2my'} \end{Bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & -1/L_1 & & & \\ & & 1/L_1 & & & \\ & & & 1 & & \\ & & & & 1 & 1/L_2 \\ & & & & & -1/L_2 \end{bmatrix} \begin{Bmatrix} F_{ix'} \\ F_{iy'} \\ M_{iz'} \\ F_{jx'} \\ F_{jy'} \\ M_{jz'} \end{Bmatrix} \quad (35)$$

It is obvious that the matrix in Equation 35 is the transpose of conversion matrix J . Thus, it can be abbreviated as

$$F_z' = J^T F_c' \quad (36)$$

The stiffness matrix in local main coordinates can be derived as follows:

$$F_z' = K_z' d_z' \quad (37)$$

$$K_z' = J^T K_c' J \quad (38)$$

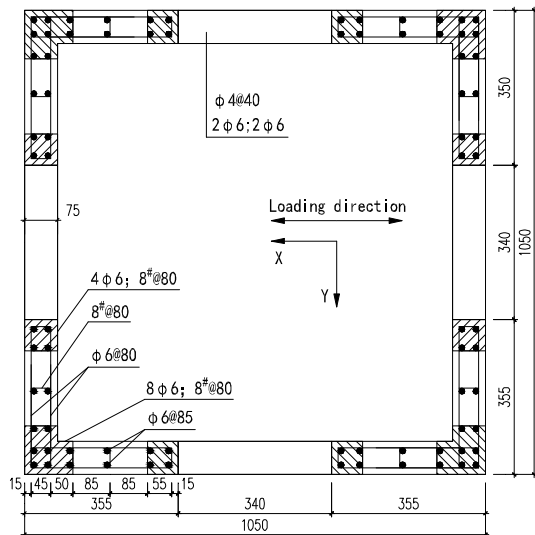
2. Global to local transformation matrix of the master degree of freedom

The displacement vector and force vector can be expressed as follows:

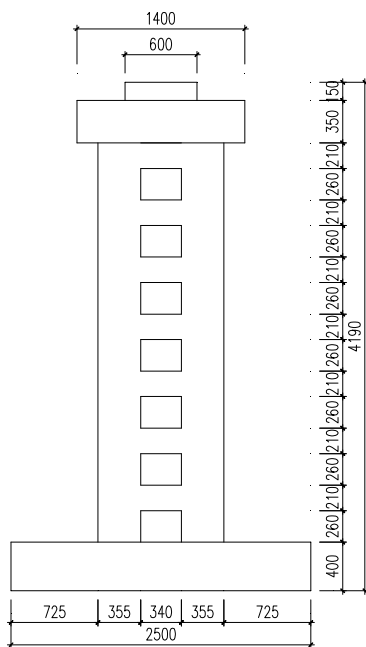
$$d_z = \{u_{1mx}, u_{1my}, u_{1mz}, u_{2qx}, u_{2qy}, u_{2qz}, u_{1qx}, u_{1qy}, u_{1qz}, u_{2mx}, u_{2my}, u_{2mz}\}^T \quad (39)$$

$$F_z = \{F_{1mx}, F_{1my}, F_{1mz}, F_{2qx}, F_{2qy}, F_{2qz}, F_{1qx}, F_{1qy}, F_{1qz}, F_{2mx}, F_{2my}, F_{2mz}\}^T \quad (40)$$

where $u_{1mx}, u_{1my}, u_{1mz}$ denote the displacement in the x, y, and z directions, respectively, on the m th node on the left wall. $F_{1mx}, F_{1my}, F_{1mz}$ denote the force in the x, y, and z directions, respectively on the m th node on the left wall. Other parameters correspond to the corresponding node displacement and node force.



(a) Cross-section steel reinforcement



(b) Dimensions (unit: mm)

Fig.6. Dimensions and Steel Reinforcement Details of the Tested RC Core Wall

The specimen is modeled by eight wall panels including four x-axis wall panels and four z-axis wall panels. Each wall panel is divided into eight wall elements along the vertical direction. The height of the lowest two wall elements is 235 mm, and the height of the other elements is 470 mm, as shown in Fig.7.

Two types of coupling beam stiffness with a different definition of rotation DOF at the beam-wall joint were used to simulate the responses of the specimen. The comparison of the predicted lateral force-top displacement curve that was obtained by two types of coupling beam stiffness with test curves is shown in Fig.8.

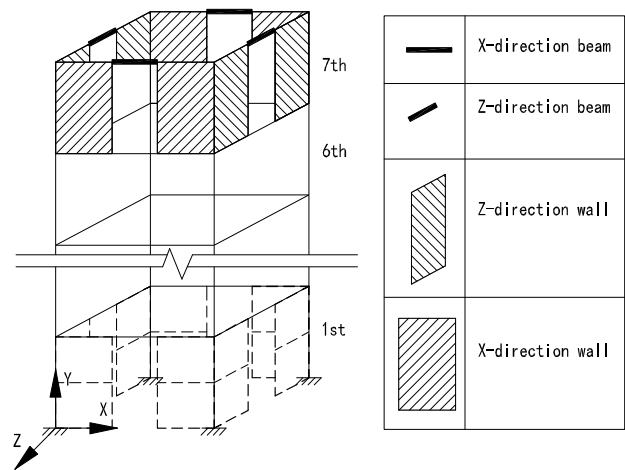


Fig.7. Wall Element Partition of Specimen

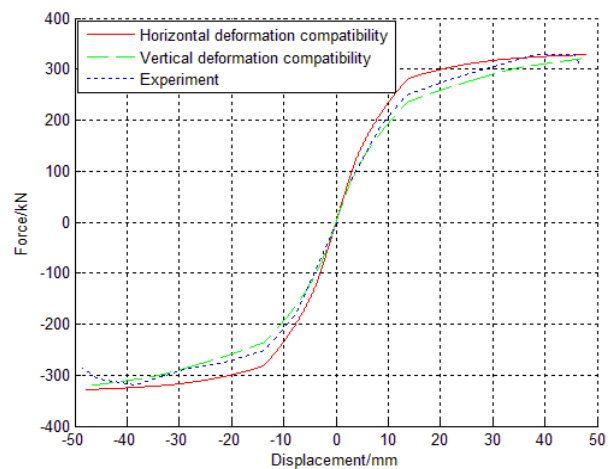


Fig.8. Comparison of Skeleton Curves with Different Deformation Compatibility Patterns

From Fig.8., it can be found that the predicted curve obtained by using the vertical deformation compatibility is much closer to the test curve.

5. Conclusions

A modified macro numerical model for coupled RC shear/core walls is proposed in this study. In this model, a three-dimensional multi-vertical-line-element model with distributed shear spring and out-of-plane freedom is adopted to simulate the wall unit, and a one-dimensional line element with rotation DOF defined on the basis of vertical deformation compatibility is adopted to simulate the coupling beam. Compared with the wall models proposed in the literature, the analytic model proposed in this study is more accurate for considering the shear deformation of the wall element, and the out-of-plane stiffness of the shear wall is also considered. Compared with the analytic models proposed by previous researchers for coupling beams, the proposed model is more accurate when considering the deformation compatibility between the beam and the wall model and is more in line with the actual situation. The accuracy of this model is verified by the case study. It is found that the predicted result

from the model with vertical deformation compatibility is more accurate than that from the model with horizontal deformation compatibility. The macro model developed in this study has advantages of reduced computation effort and high precision. It is appropriate for engineering application.

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